

Deflection by calculation

Long-term deflection from curvatures · EN 1992-1-1 §7.4.3 · UK NA / PN-EN

Reference	NAE-RC-033 · version 1.0
Standards	BS EN 1992-1-1:2004 with UK NA · PN-EN 1992-1-1:2008 with Polish NA (NDP sources listed on the NDP sheet)
National Annex	Selectable: UK NA to BS EN 1992-1-1 or Polish NA to PN-EN 1992-1-1
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What it does

Long-term deflection of a rectangular reinforced concrete beam or slab strip under the quasi-permanent load, from the curvatures of the uncracked and cracked sections with creep and shrinkage (7.4.3), against a span / ratio limit.

How to use

1. Open the Calculation sheet and select the National Annex.
2. Fill in the shaded input cells. Everything else is calculated.
3. Read the Results block; every step below it quotes its clause. Print or save as PDF for your calculation package.

Scope and limitations

Rectangular section $b \times h$ with tension steel A_s at d and optional compression steel A_{s2} at d_2 , one critical section (mid-span, or the support of a cantilever). Effective modulus $E_{c,eff} = E_{cm} / (1 + \phi)$ (7.20), ϕ and the free shrinkage strain ϵ_{cs} entered (EN 1992-1-1 Annex B, or 3.1.4 and 3.1.5; linear creep, so the stress at loading not above $0.45 f_{ck}(t_0)$, 3.1.4(4)). Section properties transformed with $\alpha_e = E_s / E_{c,eff}$, uncracked (I) and cracked (II, concrete in tension ignored). Cracking moment from f_{ctm} (7.4.3(3), (4)) on the uncracked section. The section counts as cracked when the largest moment it has carried or will carry (construction loads, frequent or characteristic service loads, entered) exceeds M_{cr} , or when it is declared cracked before (early-age loading at a lower strength and stiffness, restraint or shrinkage: the sheet does not check the cracking moment at an early age, so answer Yes when that is possible). ζ is then taken as $1 - 0.5 (M_{cr} / M_{max})^2$ (M_{max} not below M_{cr}), an approximation of ours that keeps the tension stiffening of the largest moment (7.19 itself uses the moment considered), and 0 otherwise. Compression steel lying below the cracked neutral axis is counted with α_e in tension. Curvature from the load $M / (E_{c,eff} I)$ and from shrinkage $\epsilon_{cs} \alpha_e S / I$ (7.21), each interpolated with ζ (7.18). Deflection $\delta = k L^2 (1/r)$ for the load ($k = 5/48 (1 - \beta/10)$ at mid-span of a span with end moments, $\beta = \beta_A + \beta_B$, $\beta_A = M_A / M_C$ and $\beta_B = M_B / M_C$ with the hogging end moments and the mid-span moment M_C ($\beta_A + \beta_B \leq 4$, $2 + 2$ for fixed ends), $5/48$ simply supported, $1/4$ for a cantilever under uniform load) and $kcs L^2 (1/r)$ for shrinkage ($1/8$ for a span, $1/2$ for a cantilever, uniform curvature). Along the member the load part takes the elastic shape of a uniform load with those end moments (the peak moves towards the less restrained end) and the shrinkage part that of a uniform curvature. On a continuous span the supports restrain the shrinkage curvature by an unknown amount: at every point the restrained shrinkage displacement lies between none (both ends fixed) and the simply supported value, so the larger of the displacements with and without the shrinkage part is checked. The displacement is evaluated at 201 points, with an allowance for a peak between them of a quarter of the largest second difference. Coefficients for a uniform load and a stiffness taken from the critical section: an estimate, not a member integration (7.4.3(7)). The quasi-permanent moment at the critical section is entered (ψ_2 from EN 1990). Checked as $\max |\delta| \leq L / 250$ or the ratio entered, an upward result reported as such (7.4.1(4), (5)); the part after the finishes are fixed is not separated. A rigorous integration of curvature along the member is more exact.

The second-generation EN 1992-1-1:2023 is not yet implemented.

Further reading and independent cross-check

- The Concrete Centre, How to design concrete structures using Eurocode 2: Deflection calculations.
- The Concrete Centre, Concise Eurocode 2, 10.5 (deflection by calculation).
- The Concrete Centre, RC Spreadsheets v4 (licensed product) can be used as an independent cross-check.

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About NA Engineering

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Revision history

v1.0 29 Oct 2026 · First issue

Project
Element
Date

Job no.
Made by
Checked

Inputs · member and section

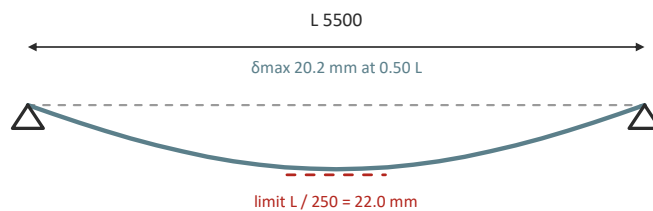
Shaded cells only

	National Annex		UK NA (BS EN)	
Table 3.1	Concrete strength class		C30/37	
	Support condition		Simply supported	
	Span (cantilever: its length)	L	5500	mm
How to: Deflection	Continuous: hogging end moment at A over the mid-span moment	θ_A	0.00	
	Continuous: the same at B MB / MC ($\beta_A + \beta_B \leq 4$; fixed ends 2 -	θ_B	0.00	
	Width	b	1000	mm
	Overall depth	h	280	mm
	Tension steel at the critical section	A_s	1131	mm ²
	Its effective depth	d	245	mm
	Compression steel (0 = none)	A_{s2}	0	mm ²
	Its depth from the compression face	d_2	35	mm
EN 1990 (6.16b)	Quasi-permanent moment at the critical section	M_{qp}	50.0	kNm
7.4.3(3)	Largest moment carried there (construction or service; $\geq M_{qp}$	M_{max}	50.0	kNm
7.4.3(3)	Cracked before (early-age loading, restraint or shrinkage)		No	
3.1.4, Annex B	Creep coefficient $\phi(\infty, t_0)$	ϕ	2.50	
3.1.4(6), Annex B	Free shrinkage strain (as a positive number, e.g. 0.0004)	ϵ_{cs}	0.00040	
7.4.1(4)	Limit span / ratio	$L / \dots$	250	
	Inputs valid and consistent			OK

Results

Long-term deflection $\max \delta / (L / \text{limit})$		0.92		OK
Largest displacement along the member (signed)	$\delta_{\max}$	20.2	mm	
Its position	x / L	0.500		
At mid-span (tip) load + shrinkage	δ	20.2	mm	
Span / largest deflection	$L / \delta _{\max}$	272		down
Cracked? (largest moment or declared)	$M_{\max} / M_{cr}$	1.10		cracked
Distribution coefficient	ζ	0.590		

Your member, elevation · quasi-permanent deflected shape (exaggerated; continuous: the larger of with and without shrinkage), the limit dashed



Calculation · materials

Table 3.1	Characteristic cylinder strength	f_{ck}	30	MPa
Table 3.1	Mean tensile strength	f_{ctm}	2.90	MPa
Table 3.1	Mean modulus $22 (f_{cm} / 10)^{0.3}$	E_{cm}	32837	MPa
(7.20)	Effective modulus $E_{cm} / (1 + \phi)$	$E_{c,eff}$	9382	MPa
3.2.7(4)	Modular ratio $E_s / E_{c,eff}$, $E_s = 200$ GPa	α_e	21.32	

Calculation · section properties (per the width b , transformed)

	Uncracked: transformed area $b h + (\alpha_e - 1)(A_s + A_{s2})$	A_I	302979	mm ²
	Neutral axis from the compression face	x_I	148.0	mm
	Second moment of area	I_I	2.063E+09	mm ⁴
	Cracked: neutral axis (A_{s2} below it: $\alpha_e A_{s2}$ in tension)	x_{II}	87.2	mm
	Second moment of area	I_{II}	8.214E+08	mm ⁴
(7.21)	First moments of the steel $A_s (d - x) - A_{s2} (x - d_2)$	S_I	109748	mm ³
(7.21)		S_{II}	178444	mm ³

Calculation · curvature and deflection

7.4.3(3), (4)	Cracking moment $f_{ctm} I_I / (h - x_I)$	M_{cr}	45.27	kNm
7.4.3	$\zeta = 1 - 0.5 (M_{cr} / \max(M_{\max}, M_{cr}))^2$ (0 if never cracked)	ζ	0.590	
(7.18)	Curvature from the load $\zeta M / (E_{c,eff} I_{II}) + (1 - \zeta) M / (E_{c,eff} I_I)$	$1/r_M$	4.888E-06	1/mm
(7.21)	Curvature from shrinkage $\epsilon_{cs} \alpha_e (\zeta S_{II} / I_{II} + (1 - \zeta) S_I / I_I)$	$1/r_{cs}$	1.279E-06	1/mm
	Coefficient for the load $(5/48 (1 - (\beta_A + \beta_B)/10)$; cantilever 1/4)	k	0.1042	
	Coefficient for shrinkage (span 1/8; cantilever 1/2)	k_{cs}	0.125	

	Deflection from the load $k L^2 (1/r)M$	δ_M	15.4	mm	
	Deflection from shrinkage $kcs L^2 (1/r)cs$ (continuous: the unres	δ_{cs}	4.8	mm	
	At mid-span (tip)	δ	20.2	mm	
	Largest $ \delta $ at 201 points (continuous: with or without shrinkag	$ \delta _{201}$	20.24	mm	
	Allowance between points $\max \text{second difference} / 4$	$\Delta\delta$	0.0012	mm	
	Largest $ \delta $ along the member	$ \delta _{max}$	20.24	mm	
	Its signed value and position x / L	δ_{max}	20.2	mm	
		x / L	0.500		
7.4.1(4)	Limit L / ratio	δ_{lim}	22.0	mm	
7.4.1(4)	$\max \delta \leq \delta_{lim}$		20.2	mm	OK

The quasi-permanent moment is the one at the critical section; for a continuous span the mid-span moment with the end moments that give θ_A and θ_B . Long-term values include the instantaneous part. Check also the span / depth rules (RC-005).

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Nationally Determined Parameters

Values used by the Calculation sheet for the selected National Annex. Sources: UK NA to BS EN 1992-1-1:2004, Table NA.1; PN-EN 1992-1-1:2008, Załącznik krajowy NA. Edit only if your project specifies different values.

Parameter	Clause	UK NA	PL NA	Notes
Partial factor, concrete γ_c	2.4.2.4(1)	1.5	1.4	Persistent & transient. UK NA: recommended; PL: NA.2, Table NA.2
Partial factor, reinforcement γ_s	2.4.2.4(1)	1.15	1.15	UK NA: recommended; PL: NA.2, Table NA.2
α_{cc} (compression in flexure / axial)	3.1.6(1)	0.85	1	UK NA: 0.85; PL: recommended (NA.9)
α_{cc} (shear and punching)	3.1.6(1)	1	1	UK NA: 1.0 for other phenomena; PL: recommended (NA.9)
α_{ct} (design tensile strength)	3.1.6(2)	1	1	Recommended value in both annexes
CRd,c numerator (CRd,c = value / γ_c)	6.2.2(1), 6.4.4(1)	0.18	0.18	Recommended value in both annexes
k1 (axial stress term, punching)	6.4.4(1)	0.1	0.1	Recommended value in both annexes
$v_{Rd,max}$ factor ($v_{Rd,max} = \text{factor} \cdot v \cdot f_{cd}$)	6.4.5(3)	0.5	0.4	UK NA: 0.5 v fcd; PL: recommended value (NA.9), 0.4 v fcd since EN 1992-1-1 AC:2010 (PN-EN 1992-1-1:2008)
kmax ($v_{Ed} \leq k_{max} \cdot v_{Rd,c}$ at u1)	6.4.5	2	1.5	UK NA 6.4.5(3): 2 vRd,c. PL: no limit in PN-EN 1992-1-1:2008; 1.5 (A1:2014 recommendation) applied conse
ϕ_{large} (large bar threshold, mm)	8.8(1)	40	32	UK NA: 40 mm; PL: 32 mm recommended (NA.9)
cot θ upper limit (variable strut)	6.2.3(2)	2.5	2	UK NA: 2.5 (1.25 with axial tension); PL: NA.4, $1 \leq \cot \theta \leq 2.0$
k1 (x/d limit, $\delta \geq k_1 + k_2 x_u/d$)	5.5(4)	0.4	0.44	UK NA for $f_{yk} \leq 500$ MPa; PL: recommended (NA.9)
k2 (x/d limit, $\epsilon_{cu2} = 0.0035$)	5.5(4)	1	1.25	UK NA: $0.6 + 0.0014/\epsilon_{cu2}$; PL: $1.25(0.6 + 0.0014/\epsilon_{cu2})$
Upper limit on the (310/ σ_s) factor (0 = none)	7.4.2(2)	1.5	0	UK NA Table NA.5 Note 5: (310/ σ_s) or (500/ f_{yk})($A_{s,prov}/A_{s,req}$) ≤ 1.5 ; PL: none
L/d not to exceed 40 K (1 = applies)	7.4.2(2)	1	0	UK NA Table NA.5 Note 6; not in EN text
ϕ_{min} (longitudinal bars in columns, mm)	9.5.2(1)	12	6	UK NA: 12 mm; PL: NA.6, 6 mm
k3 (crack spacing)	7.3.4(3)	3.4	3.4	Recommended value in both annexes
k4 (crack spacing)	7.3.4(3)	0.425	0.425	Recommended value in both annexes